Femtosecond Laser Pulse Propagation in Air

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Abstract—We present numerical results on the propagation of femtosecond pulses in ionizing air including multiphoton ionization, linear dispersion, space-time focusing and self-steepening. It is shown that all of these terms have an influence on the pulse dynamics. Specifically, we demonstrate that the self-steepening of the pulse causes a strong blue-shifted spectrum which is in qualitative agreement with the experimentally observed spectrum in air.

1. INTRODUCTION

The formation of filaments in air by the use of high-power femtosecond laser pulses has been the subject of interest both experimentally and theoretically for the past several years [1–9]. This phenomenon is still an active research area not only for its possible practical applications such as the control of lightning discharge [1] and remote sensing [5] but it is also very rich from a purely nonlinear dynamical point of view. The long-range propagation of these pulses beyond their first focus depends on the use of sub-picosecond laser pulses in which there is not enough time for optical breakdown to occur. In this case, the leading edge of the pulse creates a low density plasma which trails behind the pulse. As these pulses propagate further their spectrum broadens significantly due to self-phase modulation, turning into a white-light laser pulse. The possibility of creating a white-light laser source at a remote distance might have important applications in remote atmospheric sensing, in which different atmospheric agents could be detected simultaneously.

The underlying physical mechanism for this phenomenon can be described as a dynamical balance between focusing and refocusing by the generated plasma. This in turn creates a long filament in air. In our previous work we showed from a quasi-analytical approach how the focusing and refocusing lead to filament formation in air [9]. In general, the full dynamics of these pulses is complicated since the pulses undergo both temporal as well as spatial reshaping. Up to now all numerical simulations on the propagation of femtosecond pulses in air have been restricted to the slowly varying envelope approximation (SVEA). In this work, we go beyond the usual approximations and show that higher-order terms may become important and cannot be neglected in general. Specifically, we show that terms arising from going beyond the SVEA impact the pulse spectrum significantly, in which a much more pronounced blue-shifted spectrum with a long pedestal develops. The generalized propagation equation is derived using the approach by Brabec and Krausz [10], by including multiphoton ionization of the air molecules. A similar propagation equation including multiphoton and avalanche ionization has been used by Gaeta [11] to describe the white-light generation in solid materials.

2. MODEL

Here, we consider the propagation of a linearly polarized laser pulse in ionizing air. The underlying propagation equation beyond the SVEA can be written in dimensionless form in the retarded coordinate frame ($\tau = t - z/v_p$) as:

$$
\left\{ i \frac{\partial}{\partial \tau} + \frac{1}{4} \left( 1 - is \frac{\partial}{\partial \tau} \right) \nabla_\bot^2 - \frac{L_{\text{dif}}}{4L_{\text{dis}} \omega^2} \right\} \mathcal{E}(r, z, \tau) 
+ \left( \frac{L_{\text{dif}}}{L_{\text{olu}}} \right) \left( 1 + is \frac{\partial}{\partial \tau} \right) |\mathcal{E}|^2 \mathcal{E} 
- \frac{L_{\text{dif}}}{L_{\text{plasma}}} \left( 1 - is \frac{\partial}{\partial \tau} \right) N_e \mathcal{E} + i \frac{L_{\text{dif}}}{L_{\text{MPA}}} \mathcal{E}^{2n+2} = 0
$$

Here, the generated electron density $N_e(z, r, \tau)$ appearing in Eq. (2) is obtained from

$$
\frac{\partial N_e}{\partial \tau} = |\mathcal{E}|^{2n}
$$

$\mathcal{E}(z, r, \tau)$ is the electric field envelope function and

$$
\nabla_\bot^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}
$$

We assume an initially collimated
Gaussian pulse $\mathcal{E}(z = 0, r, \tau) = e^{-r^2 - \tau^2}$, where $\mathcal{E}$ is normalized to the peak input field. The transverse $r$ and temporal $\tau$ coordinates are normalized to the initial beam radius $w_0$ and pulse width $\tau_0$ respectively. The propagation distance $z$ is given in units of the diffraction length $L_{\text{dif}} = k w_0^2 / 2$. The dispersion length scale is given as $L_{\text{dis}} = \tau_0^2 / 2 k''$, where $k''$ is the group-velocity dispersion coefficient which is $k'' = 0.2 \text{ fs}^2 / \text{cm}$ at $\lambda_0 = 800 \text{ nm}$ for ordinary air. The nonlinear length scale $L_{\text{nl}} = 1 / n_2 k_0 I_0$, where $n_2 = 4 \times 10^{-19} \text{ cm}^2 / \text{W}$ [12], is the nonlinear index of refraction and $I_0$ is the input intensity given in units of W/cm$^2$. $L_{\text{nl}}$ can also be written as $L_{\text{nl}} = 2 P_0 / P_{\text{cr}}$, where $P_0$ is the input power and $P_{\text{cr}} = \lambda_0^2 / 2 \pi n_0 n_2$ is the critical power for self-focusing in the CW limit, which is about 3 GW for air at 800 nm. The generated electron density $N_e$ is normalized to $N_e = 3 \times 10^{19} \text{ cm}^{-3}$, where $N_0 = 3 \times 10^{19} \text{ cm}^{-3}$ is the neutral number density of air molecules (%20 O$_2$ and %80 N$_2$) and $\sigma^{(n)}$ is the ionization cross section. The ionization rate is obtained by fitting the experimental data to the form of $\sigma^{(n)} I$. The other length scales are defined as: $L_{\text{plasma}} = km_e e^2 / 2 \pi n_2 k_0 N_0 \sigma^{(n)} I_0^2$, $s = 1 / \omega_0 \tau_0$, and $L_{\text{MPA}} = nh \omega_0 N_0 \sigma^{(n)} I_0^2 / 2$. 

**Fig. 1.** Shown is the spatio-temporal intensity distribution of an initial Gaussian pulse propagating in ionizing air at (a) $z = 0.4$ and (b) $z = 1.0$ in units of the diffraction length $L_{\text{dif}}$. Initially, a sharp pulse develops towards the leading part of the pulse with a smoother pulse at the trailing edge of the pulse (a). Upon further propagation due to dispersion and self-steepening effects the front part of the pulse diminishes where most of the pulse energy is pushed towards the trailing part of the pulse. The shock formation at the trailing part of the pulse (b) is due to the self-steepening of the pulse.
3. RESULTS AND DISCUSSION

We integrate Eqs. (2) and (3) with the initial conditions $\delta_{\text{FWHM}} = 150\text{ fs}$, $w_0 = 250\text{ \mu m}$ and $P_0 = 6P_{\text{cr}}$. In experimental conditions however, a much larger beam diameter is used which is numerically difficult to handle due to the fixed grid used in the numerical scheme. In order to further mimic the experimental conditions we use $L_{\text{dif}}/L_{\text{dis}} = 0.05$. A quantitative comparison is not expected but as we will show later that most of the results are in qualitative agreement with the experiment. For $s = 0$, Eq. (2) has been studied numerically before [4, 6, 7]. Here, we do not include the effect arising from a delayed Kerr nonlinearity for simplicity. In
pulse, however other slices that are above the critical focusing events. Shown here is only one slice of the pulse, however other slices that are above the critical power for self-focusing exhibit similar behavior, manifesting the overall dynamics of the pulse much more complex. The refocusing phenomenon can also be seen in Fig. 4, where the ratio of filament energy defined as

$$E_{\text{fil}}(z) = 2\pi \int_0^1 I(z, r, \tau) r dr d\tau$$

to the total pulse energy is plotted as a function of propagation distance. As the pulse focuses the pulse contracts and consequently more energy is fed into the core of the beam until refocusing takes over diffractioning the beam. In this case less energy is contained in the fixed core region. However, the beam defocuses to a point where focusing takes over again, and the filament energy increases. This picture is consistent with the previous illustration in Fig. 2 where refocusing occurs around $z = 0.55$. Refocusing was demonstrated experimentally at Laval University, through the measurement of the filament energy [4] and average filament peak intensity [8]. In Fig. 5 the on-axis total electron density is plotted as a function of propagation distance. The dynamics of the electron density reflects the fact that the pulse undergoes multiple focusing and defocusing events. It is clear from numerical simulations that the overall dynamics of such pulses is rich and complex. To further elucidate the dynamics of the pulse, in Fig. 6 the fluence of the pulse defined as $J(z, r) = \int_{-\infty}^{\infty} I(z, r, \tau) d\tau$, (which is experimentally accessible), is plotted for two different propagation distances. It can be seen that initially a dip occurs in the center of the fluence which vanishes at later distance and does not change significantly after that, giving the appearance of a self-guided pulse.

4. CONCLUSION

In conclusion, we have shown that for femtosecond pulses propagating in air the SVEA breaks down, and that the strongly blue shifted spectrum is due to the self-steepening of the pulse. In experiments white-light generation occurs after the pulse has propagated significantly, a fact observed in our numerical simulations as well.

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REFERENCES


