Local stress distribution on the surface of a spherical cell in an optical stretcher

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Abstract: We calculate stress distribution on the surface of a spherical cell trapped by two counter-propagating beams in an optical stretcher in the ray optics regime. We explain the apparition of peaks in the stress distribution, which were not revealed in the earlier published results. We consider the divergence of the incident beams from the fibers, and express the stress distribution as a function of fiber-to-cell distance. In an appendix, we show that the local scattering stress is perpendicular to the spherical refractive surface regardless of incident angle, polarization, the reflectance and transmittance at the surface. Our results may serve as a guideline for the optimization of experimental parameters in optical stretchers.

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References and Links


1. Introduction

The ability of cells to sense an external tension and to react to the physical stress is intimately related to their functions and properties. In this context, morphological deformations of yeast cells [1], red blood cells (RBCs) [2], and Chinese hamster ovary (CHO) cells [3] under external optical forces have been studied with dual-beam fiber-optical stretchers consisting of...
two non-focused counter-propagating laser beams from two well-aligned single-mode optical fibers. In an optical stretcher, the cell is confined on the common optical axis of the two beams by the transverse gradient forces, and is stabilized at a point on the optical axis where the two laser beam scattering forces balance each other. As the index of refraction inside the cell is usually greater than that of the surrounding medium, the changes in the photon momentum due to Fresnel reflection and refraction at the interface tend to stretch the cell. The optical stretcher can potentially allow us to measure the elasticity of cells, to sort cells by their membrane viscoelastic properties and to detect differences (or malignancy) in cell’s structures for early cancer detection. In these applications, the optical stretcher applies forces that are less localized, smoother and with less disturbance to the cells, in comparison with other approaches such as optical tweezers with beads attached to the cells [4-6], atomic force microscope [7] or micropipette [8] so that the elasticity of the whole cell is measured in a more natural environment.

To study the cell’s response to external forces in an optical stretcher, the precise stress distribution on the cell surface needs to be determined. The studies on the photonics radiation pressure of a laser trap on a dielectric sphere in the ray optics regime by Ashkin and many authors [9-11] are well known. Most works model the trapped particle as a tiny rigid non-deformable sphere. Consequently, only the total force applied to the trapped object is of interest. However, in the dual-beam optical stretcher applications, we are interested in the local stress distribution on the cell’s surface and in the concomitant cell’s deformation [2]. Guck gave the first comprehensive but approximate theory on the photonics stress distribution in the dual-beam fiber-optic stretcher on a spherical deformable red blood cell. In this paper, we present a more precise theory of the scattering stress distribution on the surface of a spherical cell. Our results show local peaks in the stress profile that were not previously outlined. The peaks result from the focusing power of the spherical cell acting as a thick convergent lens. An analytical proof is given in the appendix to show that the local scattering stress is perpendicular to the spherical surface, regardless of the incident angle, polarization of the incident beam, as well as the reflectance and the transmittance at the cell surface. In addition, we consider the divergence of the beams from the fibers, and express the stress distribution as a function of fiber-to-cell distance, which is directly measurable in experiments, for a given fiber numerical aperture.

2. Theory

Consider a spherical cell trapped in a dual-beam fiber-optical stretcher. Cells do not usually have a spherical shape. However, with red blood cells (RBCs) one can dilute the buffer to have the cell osmotically swollen to a spherical shape. Then, the RBC is modeled as a spherical elastic membrane shell filled with homogeneous and isotropic fluid [2]. Experimentally, near-infrared light is often used to minimize the damage to the cell caused by light absorption [12]. We use $\lambda=1.064\mu m$ in our calculation. For a RBC cell of radius around $\rho=3\mu m$, the ray optics regime criterion ($2\pi\rho/\lambda >>1$) is satisfied. Any near-infrared wavelength can be used in our theory. Only the calculated results are expected to change accordingly.

When a laser beam encounters a dielectric interface, scattering forces are generated by the change in photon’s momentum $P=n_\text{E}E/c$ [9], where $n_\text{E}$ is the refractive index of the buffer medium, $E$ is the beam energy and $c$ the speed of light. We denote the momentum of the incident, transmitted and reflected rays by $\vec{P}_i$, $\vec{P}_t$ and $\vec{P}_r$, and their directional unit vectors by $\hat{\vec{a}}_i$, $\hat{\vec{a}}_t$ and $\hat{\vec{a}}_r$ respectively. Then, according to the law of momentum conservation, the stress $\sigma$ applied to the cell’s refractive surface is expressed as:

$$\Delta \vec{P} = \vec{P}_i - (\vec{P}_t + \vec{P}_r)$$

$$\Delta \vec{P} = \frac{\vec{P}_i - (\vec{P}_t + \vec{P}_r)}{A} = \frac{1}{c} \frac{E}{A} \eta (\hat{\vec{a}}_i - \hat{\vec{a}}_t - \hat{\vec{a}}_r) = \frac{\eta}{c} \frac{\partial F}{A} Q$$  \hspace{1cm} (1)
where $P$ is the laser beam power, $A$ the area covered by the beam, $n = n_2/n_1$ with $n_1$ and $n_2$ being the index of the medium surrounding the cell and inside the cell, respectively, $T$ and $R$ are the Fresnel transmittance and reflectance, respectively, and $\mathcal{Q}$ is the dimensionless momentum transfer vector defined by Eq. (1). The problem is thus reduced to finding $\mathcal{Q}$. For a stable trap [10], the ratio of the full width at half maximum (FWHM) of the laser beam $w$ over the radius of the cell $\rho$, $(w/\rho) > 1$. In Ref. [2] the incident rays are assumed parallel to the optical axis (denoted as the $x$-axis in Fig. 1). We consider a diverging Gaussian beam from the fiber hitting the front surface (left half) of the sphere. For a given distance from the fiber end to the cell center $D$, there is a unique relationship between the incident point defined by polar angle $\phi_1$ and the incident angle $\varepsilon$. In fact, as shown in Fig. 1 $\phi_1 = \varepsilon - \delta$ with $\delta = \tan^{-1}(\rho \sin \phi_1/(D - \rho \cos \phi_1))$. The polar angle $\phi$ is the incident angle when the beam is parallel to the $x$-axis.

![Fig. 1. Incident, reflected and transmitted rays on a spherical object.](image)

The refraction angle $\beta$ is determined by the Snell’s law $n_1 \sin \varepsilon = n_2 \sin \beta$. After the first refraction, the transmitted ray hits the back surface (right-half) of the sphere from the inside of the cell at a point defined by polar angle $\phi_2 = 2\beta - \phi_1$. The angle of the reflected ray to the $x$-axis is $\pi - (3\beta - \phi_1)$, and that of the transmitted ray is $\varepsilon + \phi_1 - 2\beta$, as shown in Fig. 1. As the reflectance is on the order of $10^{-3}$ at normal incidence for the refractive index $n_1 = 1.33$ for the buffer, and $n_2 = 1.38$ for the cells, the third and subsequent reflections inside the cell would have relatively weak power and result in weak stress, which can be neglected. Once all angles are found, one can deduce $\mathcal{Q}$. For the front surface, we have:

\[
Q_{\text{front } X} = \exp\left[-2(\rho \sin \phi_1/w)^2 \left[\cos(\delta) - n_1 T(\varepsilon) \cos(\phi_1 - \beta) + R(\varepsilon) \cos(2\varepsilon)\right]\right]
\]

\[
Q_{\text{front } Y} = \exp\left[-2(\rho \sin \phi_1/w)^2 \left[\sin(\delta) + n_1 T(\varepsilon) \sin(\phi_1 - \beta) - R(\varepsilon) \sin(2\varepsilon)\right]\right]
\]

For the back surface we have:

\[
Q_{\text{back } X} = \exp\left[-2(\rho \sin \phi_1/w)^2 T(\varepsilon)\left[n_1 \cos(\phi_1 - \beta) + n_2 R(\varepsilon) \cos(3\beta - \phi_1) - T(\beta) \cos(\phi_1 - 2\beta)\right]\right]
\]

\[
Q_{\text{back } Y} = \exp\left[-2(\rho \sin \phi_1/w)^2 T(\varepsilon)\left[-n_1 \sin(\phi_1 - \beta) + n_2 R(\varepsilon) \sin(3\beta - \phi_1) + T(\beta) \sin(\phi_1 - 2\beta)\right]\right]
\]

where $\exp\left[-2(\rho \sin \phi_1/w)^2\right]$ is a Gaussian beam correction factor. One should note that we made no assumption thus far about the state of polarization, which can affect the reflectance and the transmittance.

It is interesting to prove that the scattering stress is always perpendicular to the spherical refraction surface regardless of the incident angle and no matter whether the rays hit the
surface from outside or inside the sphere; i.e. both \( \hat{Q}_{\text{front}} \) and \( \hat{Q}_{\text{back}} \) are perpendicular to the spherical surface. In the appendix we analytically prove, using Eq. (1) and Eqs. (2-5), that:

\[
\arctan \frac{Q_{\text{front} Y}(\phi_1)}{Q_{\text{front} X}(\phi_1)} = \phi_1 \quad \text{and} \quad \arctan \frac{Q_{\text{back} Y}(\phi_1)}{Q_{\text{back} X}(\phi_1)} = 2\beta - \phi_1 = \phi_2
\]  

(6)

Note that this proof is independent of the Fresnel reflectance \( R \) and transmittance \( T \) and therefore of the incident beam polarization. With the proof of the orthogonality we can write:

\[
\begin{align*}
Q_{\text{front} X} &= \exp\left[-2\left(\rho \sin(\phi_1) / w\right)^2\right]Q_{\text{front} \cos(\phi_1)} \\
Q_{\text{front} Y} &= \exp\left[-2\left(\rho \sin(\phi_1) / w\right)^2\right]Q_{\text{front} \sin(\phi_1)} \\
Q_{\text{back} X} &= \exp\left[-2\rho^2 \sin^2(\phi_1) / w^2\right]Q_{\text{back} \cos(2\beta - \phi_1)} \\
Q_{\text{back} Y} &= \exp\left[-2\rho^2 \sin^2(\phi_1) / w^2\right]Q_{\text{back} \sin(2\beta - \phi_1)}
\end{align*}
\]  

(7)

Equations (7) were given in Ref. [2] without Gaussian beam correction and have been implied without proof that the stress is perpendicular to the spherical surface. For the sake of simplicity, a random polarization is usually considered [2,9] and the average reflectance and transmittance of the parallel and perpendicular polarizations are used. In that case, no parameter in Eqs. (4-7) varies with the meridional angle, so that the three dimensional trapping system is rotationally symmetric around the \( x \)-axis and can be analyzed in the \( x-y \) plane only.

It is interesting to examine the output polar angle, \( \phi_2 = 2\sin^{-1}\left[\frac{n_1}{n_2}\sin\varepsilon\right] - \varepsilon + \delta \), where \( n_1 < n_2 \), as a function of the incidence angle \( \varepsilon \). For small \( \varepsilon \), \( \phi_2 \) increases with \( \varepsilon \) monotonically. Then, the increase of \( \phi_2 \) is slowed down and finally \( \phi_2 \) decreases with increasing \( \varepsilon \) for \( \varepsilon > \tilde{\varepsilon} \), as shown in Fig. 2, where \( \tilde{\varepsilon} \), corresponding to a maximum output polar angle \( \phi_2 \), can be computed by the derivative of \( \phi_2 \) with respect to \( \varepsilon \). Consequently, there is an upper limit of the output polar angle \( \phi_2 \) that depends on the indices \( n_1 \) and \( n_2 \), the ratio \( w/\rho \), the fiber NA and the cell radius \( \rho \). As an example, for \( NA=0.11, n_1=1.335, n_2=1.37, D=39.9\mu m \) (\( w/\rho=1.1 \)) and \( \rho=3\mu m, \phi_2=71^\circ \). There is no incident ray at the front surface, whose refracted ray can hit the back surface at a position of polar angle higher than the upper limit. Below the upper limit there is a range of \( \phi_2 \), where the same output position \( \phi_2 \) can be reached by two different incident angles, as shown in Fig. 2(a). This range is \( 65^\circ \leq \phi_2 \leq 71^\circ \) in our example.

![Fig. 2. (a) Position of the output ray on the back surface as a function of incidence angle. Intersection of the horizontal lines and curves are the solutions for two incident rays at a same output position. In the example shown in (b), at incident angles \( \varepsilon=78.2^\circ \) and \( 84.3^\circ \) the rays pass through the cell and hit the back surface at the same point with a polar angle of \( \phi_2=70^\circ \). (b) In the dual-beam optical stretcher the two counter-propagating laser beams generate two stress distributions, which are added up. In the example shown in Fig. 2(b) on the back surface and at the polar angle position \( \phi_2=70^\circ \) the cell is also hit by a third incident ray from the counter-propagating laser beam. At that position, the contributions of the three rays should be](image)
added up. One can therefore separate both the front and back surfaces into 4 regions. In the first region, one incident ray from outside and one ray from inside the cell hit the same point on the surface. This region is limited by the polar angle $0^\circ<\phi<65^\circ$ in our example. In the second region, three rays, one incident ray from outside and two rays from inside of the cell hit the same point at the surface. This region is limited by the polar angle $65^\circ<\phi<71^\circ$ in our example. In the third region only one external ray hits the cell and exerts a stress on the cell surface, and no other ray can hit this point from inside the cell. The third region is limited by the polar angle $71^\circ<\phi<87^\circ$ in our example. The fourth region is limited by the highest position that a ray can hit on the surface for a given fiber $NA$ and fiber-to-cell distance $D$. Thus, no stress is applied for $\phi>87^\circ$ in the example. In the first and second regions, to calculate the stress applied to the front surface at a position $\phi_1$, we need to find the incidence angle of the ray ($\varepsilon_2$) and $\phi_2(\varepsilon_2)$ coming in the $-x$ direction, which will make a polar angle of $\phi_1=(2\beta-\phi_2)$ to the $-x$ direction when hitting the position $\phi_1$. In other words, we need to solve equation $2\beta-\phi_2(\varepsilon_2)=\phi_1(\varepsilon_1)$ to find $\phi_1$ for a given $\phi_2$. Then we compute Eqs. (2-3) with $\phi_1$, Eqs. (4-5) with $\phi_2$ and we add the results using Eq. (8):

$$Q_{\text{tot}} = Q_{\text{front}} + Q_{\text{back}} = \sqrt{Q_{\text{front}}x(\phi_1)^2} + \sqrt{Q_{\text{front}}y(\phi_1)^2} + \sqrt{Q_{\text{back}}x(\phi_2)^2} + \sqrt{Q_{\text{back}}y(\phi_2)^2}$$  (8)

A sketch of $Q_{\text{back}}$ and $Q_{\text{front}}$ is depicted in Fig. 3, where the front surface is denoted as the first surface and the back surface as the second for the incident beam in $+x$ direction, and vice versa for the incident beam propagating in $-x$ direction. Indeed, in the region $0^\circ<\phi<65^\circ$ the stress is the addition of contributions of the two rays. In the region $65^\circ<\phi<71^\circ$, the stress is the addition of contributions of the three rays and is thus much more intense. In the region $\phi>71^\circ$ only the rays hitting the surface from outside of the cell contribute. Finally, in the region $\phi>87^\circ$ the stress profile at the first surface is cut to zero.

![Fig. 3 Stress profile as function of the polar angle. Thick line: at the second surface; Thin line: at the first surface. NA=0.11, n_1=1.335 and n_2=1.37, D=39.9\text{\mu m} (w/\rho=1, l) and \rho=3\text{\mu m}.](image)

The total stress is the sum of those applied at the first and second surfaces, shown by the two curves in Fig. 3. The stress distribution, which is symmetric in each quadrant of Fig. 1, is shown in Fig. 4 for different distances $D$ from the fibers end to the cell center. We can see the peaks in the stress distribution located at about $60^\circ$, $120^\circ$, $240^\circ$ and $300^\circ$ positions, which were not revealed in the earlier results [2]. We anticipate that this difference may influence the predicted cell deformation.

![Fig. 4 Stress profile (Nm\textsuperscript{-2}) for different distances D with $\beta_0^2=100\text{mW}$, $\rho=3\text{\mu m}$, $n_1=1.335$, $n_2=1.37$ and NA=0.11.](image)
At small incident angles, close to 0° and 180°, for a given cell radius ρ the stress strength is proportional to the intensity of the input beam, which decreases as 1/D^2. However the Gaussian beam correction factor exp[-2ρ^2sin^2(φ)/w^2] increases as the beam size w or the distance D increases. This is why at greater incident angles, close to 90° and 270°, the stress profiles are less sensitive to the distance. We see in Fig. 4 that the surface region near 90°, where no stress is applied, gets smaller as the distance D increases, and that the width of the peak stays almost the same at all distances.

3. Conclusion

Neglecting the subsequent reflections inside the spherical cell, we have expressed the optical stress distribution as a function of the fiber-to-cell distance, which is directly measurable in experiments for a given fiber NA. We have shown that the focusing power of the spherical cell concentrates the refracted rays to a smaller area on the second interface, resulting in peaks on the stress distribution around certain angular positions. In addition, we have demonstrated that the optical stress is perpendicular to the spherical surface independently on the incident angle, polarization of the incident beam, reflectance and transmittance at the cell surface.

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Appendix

For an arbitrary ray incident to the surface at a point of polar angle φ and having an incident angle ε, we can rotate the x-y axes by δ=ε−φ (see Fig. 1) such that the incident ray is parallel to the new x-axis. Thus, the incident angle is equal to the polar angle ε=φ with the new axes. We consider G=Q_front y/Q_front x. Using the Snell’s law n=n_2/n_1= sin(φ)/sin(β) and redistributing the parentheses, we obtain

\[ G = \frac{\sin(\phi)T[\sin(\phi)\cos(\beta) - \cos(\phi)\sin(\beta)] - 2\cos(\phi)\sin(\beta)\sin(\phi)}{-T\sin(\phi)\cos(\phi)\cos(\beta) + [-T\sin^2(\phi)\sin(\beta) + T\sin(\beta) - 2T\cos^2(\phi)\sin(\beta)] + 2\cos^2(\phi)\sin(\beta)} \]  

(A1)

where the bracketed term in the denominator reduce to –Tcos^2(φ)sin(β). Equation (A1) can then be simplified to obtain A2:

\[ G = \frac{\sin(\phi)T[\sin(\phi)\cos(\beta) + \cos(\phi)\sin(\beta)] - 2\cos(\phi)\sin(\beta)\sin(\phi)}{-\cos(\phi)T[\sin(\phi)\cos(\beta) + \cos(\phi)\sin(\beta)] - 2\cos(\phi)\sin(\beta)} = \frac{\sin(\phi)}{-\cos(\phi)} \]  

(A2)

therefore \arctan(G)=−φ, where the negative sign means the stress is in +y direction and –x direction for 0<φ<π/2, i.e. the stress is directed away from the cell, and thus stretching the cell.

For the proof of the second part of Eq. (6), we consider G=Q_back y/Q_back x and we use φ_2=2β−φ to express the angles. We have:

\[ G = \frac{-n\sin(\beta - \phi_2) + n(1 - T)\sin(\beta + \phi_2) + T\sin(\phi - \phi_2)}{n\cos(\beta - \phi_2) + n(1 - T)\cos(\beta + \phi_2) - T\cos(\phi - \phi_2)} \]  

(A3)

Using the sine and cosine laws, we rewrite:

\[ G = \frac{2n\cos(\beta)\sin(\phi_2) - nT[\sin(\beta)\cos(\phi_2) + \cos(\beta)\sin(\phi_2)] + T\sin(\phi)\cos(\phi_2) - T\cos(\phi)\sin(\phi_2)}{2n\cos(\beta)\cos(\phi_2) + nT[\cos(\beta)\cos(\phi_2) - \sin(\beta)\sin(\phi_2)] - T\cos(\phi)\cos(\phi_2) + T\sin(\phi)\sin(\phi_2)} \]  

(A4)
and using the Snell’s law, A4 reduce to $G = \sin(\phi_2)/\cos(\phi_2)$. Again, $\arctan(G) = \phi_2 = 2\beta - \phi$. The positive sign meaning the stress is in $+y$ direction and $+x$ direction for $0 < \phi < \pi/2$. Thus, the local scattering force is perpendicular to the spherical refraction surface.