Rigorous solution for the transient surface plasmon polariton launched by subwavelength slit scattering

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Abstract: We demonstrate by a rigorous theoretical calculation the transient Surface Plasmon Polariton (SPP) mode, which is launched by a beam impinging onto a sub-wavelength real metallic slit in addition to the conventional long-range SPP. Different from the previous works, we find a direct closed-form solution of the Maxwell’s equations by the Sommerfeld branch-cut integrals without approximation. The transient wave is a SPP with a complex-valued envelope, expressed as the exponential integral. Its rapid damping may be asymptotically approximated as \(-\ln(x), 1/\sqrt{x}\) and \(1/x\), respectively, depending on the distance range away from the slit. The transit SPP may be considered as a cylindrical wave, which is radiated from the slit, takes the SPP propagation constant at the interface and propagates with additional drop due to the loss of energy pumped to the SPP.

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References and links
10. See Section 4 in Ref [8].
1. Introduction

Scattering of light by a subwavelength metallic aperture is a fundamental problem in the theory of near-field diffraction. Recent experiment revealed the existence of a short-range transient regime, in which the surface waves have a rapid drop in amplitude within first few micrometers from the groove before reaching a long-range regime [1]. This experiment involved the interference between the surface waves launched from a subwavelength groove and the transmitted light through a neighboring slit, which has been proved experimentally [2,3] and by Finite Difference in Time Domain (FDTD) calculation [4]. The scalar Composite Diffracted Evanescent Waves (CDEWs) model [2] based on the formulation of Kowarz [5] predicted an amplitude drop as \(1/x\) with the distance \(x\) from the groove. Lalanne et al. suggested that the launched surface waves consist of a long-range surface plasmon polariton (SPP) mode and a short-range “creeping wave” described by a branch-cut integral, which was computed numerically [6]. Ung et al. proposed an asymptotic closed-form solution using the modified steepest descent method, showing that the short-range surface waves have free-space cylindrical property and damp as \(1/\sqrt{x}\) [7]. Leveque et al. used a model by artificially inserting two poles in the CDEW formulation in order to include the SPP description in the theory [8]. This empirical manipulation based on physical intuition is however not theoretically and rigorously valid.

Among the numerical calculations [2,6-9] the best fit to the experimental data is given by the Green’s tensor technique [8]. However a closed-form solution in general would describe the physical phenomenon in a better and simpler way. In this paper we report a rigorous full analytical approach by solving the Sommerfeld branch-cut integrals. Some mathematical details of the solutions for the branch-cut integrals are described. Our rigorous and formally correct solution shows that the surface waves launched by scattering of a real-metal subwavelength slit are a sum of a conventional SPP and a spatial transient SPP that has a complex-valued envelope described by the exponential integral. The exponential integral envelope of the transient SPP was obtained by the empirical approach in [8]. However, their solution includes only the evanescent part and excludes the contribution of the homogeneous modes at the surface. As a result, the hard-cut in the Fourier spectrum leads to strong oscillatory behavior of the envelope, which was explained as a beating between the two envelopes predicted by their formulation [10]. Such a beating does not exist in our result. Our solution is rigorous and allows quantifying the damping of the transient SPP as asymptotical approximate by \(-\ln(x), 1/\sqrt{x}\) and \(1/x\), respectively, depending on the distance ranges away from the slit. This result fills an important missing part in the literature, since no rigorous full analytical solution on the transit SPP yet exists before [11].

2. Maxwell’s equations

Consider a real metal thick film perforated by an infinitely long slit along the \(z\)-axis with a sub-wavelength extent in the \(x\)-direction. A TM-polarized beam with the magnetic field parallel to the \(z\)-axis impinges normally on the slit. It has been demonstrated by numerical solutions of the Maxwell equations using the FDTD that the scattering by the slit induces accumulated oscillating electric charges of opposite polarity at the slit’s two corners, emulating a Hertzian electric dipole [12,4]. Thus, the scattered field may be calculated as a radiation from an electric dipole, which replaces the slit [4,8]. The electromagnetic field over a conductor-dielectric interface induced by a vertical or horizontal electric dipole has been analyzed since a long time in the cases of microwaves [13,14]. Most computations are for 3D geometries. We adopt the model with an equivalent magnetic line-source as in Ref. [6], which in the 2D case leads to a simple formula solution.

We express the electric dipole source by an equivalent magnetic type source. As a magnetic dipole can be expressed by a circulating electric current, the electric dipole in \(x\) direction can
be also expressed as a circulation of equivalent magnetic current, which is in the 2D case a line source oscillating in the \( z \) direction for the TM field. However, such a magnetic current has no physical interpretation, because the magnetic charges do not exist. This non-physical equivalent source allows us to compute the magnetic field by the inhomogeneous Maxwell’s equations. The oscillating magnetic current along the \( z \)-axis is described by the current density vector \( J_m^* \delta(y + d) \delta^z \), where \( J_m^* \) is used to indicate that \( J_m^* \) is a non-physical equivalent source and \( d \) is the offset distance of the line source to the interface, as shown in Fig. 1. The Maxwell’s equations in equivalent magnetic current formalism are:

\[
\nabla \times E_j = -\mu_0 \partial_t H_j; \\
\nabla \times H_j = -\varepsilon_j \partial_t E_j,
\]

where the subscript \( j = 1, 2 \) denotes the region of metal and dielectric, respectively. Those equations lead to, for the temporal harmonic \( e^{-i\omega t} \) and TM field, the Helmholtz equation in the spatial Fourier transform domain [14] (see Appendix A):

\[
\left( \partial_y^2 + \gamma_j^2 \right) \tilde{H}_j = -i \omega \varepsilon_j J_m^* \delta(y + d),
\]

where \( \tilde{H}_j \) is the Fourier transform of the magnetic intensity field with respect to \( x \) with the spatial frequencies \( k_0 \beta \) in the \( x \)-direction and \( \gamma_j^2 = k_0^2 (\varepsilon_j - \beta^2) \). The offset \( d \) of the line source position is introduced to avoid the discontinuities related to the source on the interface. Once the boundary conditions are satisfied, the offset will be set to zero. The solution of Eq. (1) in region 1 (metal), where the source is included, is composed of the general and particular solutions. The general solution is a superposition of planes waves and the particular solution is given by the 1D Green’s function \(-\varepsilon_j \omega J_m^* \exp(i\gamma_j \{y + d\}) / 2\gamma_j \). In region 2 (dielectric), we have the general solution only. Applying the boundary conditions at the interface to the solutions in both regions, we obtain the frequency distribution \( \tilde{H}_j \) (see Appendix B). As we are interested only in the field at the surface, we set \( y = 0 \), take \( d = 0 \) and perform the inverse Fourier transform to obtain the magnetic intensity component \( H_z \) at the interface, which is expressed as a Sommerfeld-type integral:

\[
H_z(x) = -\frac{\omega J_m^*}{2\pi} \int_{-\infty}^{\infty} \frac{\varepsilon_m \varepsilon_d \exp(ik_0 \beta x)}{\varepsilon_d \sqrt{\varepsilon_m - \beta^2} + \varepsilon_m \sqrt{\varepsilon_d - \beta^2}} d\beta,
\]
3. Sommerfeld integrals

To compute the integral Eq. (2), we choose a contour of integration closed in the up-half plane of $\beta$, $\text{Im}\{\beta\} \geq 0$, with a half-circle that encircles two branch-cut points $\beta = \varepsilon_m^{1/2}$ for the metal medium and $\beta = \varepsilon_d^{1/2}$ for the dielectric medium, and a pole whose location is determined by setting the denominator of the integrant equal to zero:

$$\beta_p = \sqrt{\varepsilon_m \varepsilon_d / (\varepsilon_m + \varepsilon_d)} = k_{sp} / k_0 \ ,$$

where $k_{sp}$ is the SPP propagation constant according to the dispersion relation. The contribution of the half-circle to the integral vanishes when its radius tends to infinity. The contribution of the pole is calculated via the residue as:

$$H_{sp} = i2\pi e^{ik_{sp} \sqrt{\varepsilon_m \varepsilon_d} / (\varepsilon_d + \varepsilon_m^{1/2})} \ ,$$

which is the conventional SPP at the interface.

From Cauchy’s integral formula, we write:

$$H_{sp}(x) = \frac{\omega J_m}{2\pi} \left( H_{sp} + \int_{r_m} f(\beta)d\beta + \int_{r_d} f(\beta)d\beta \right) ,$$

with $f(\beta)$ denoting the integrant in Eq. (2). The two branch-cut integrals around the points $\varepsilon_m^{1/2}$ and $\varepsilon_d^{1/2}$ have been considered as representing the near-field creeping waves in Ref. [6]. We compute the branch-cut integral around $\beta = \varepsilon_m^{1/2}$ first. The second branch-cut integral around $\beta = \varepsilon_d^{1/2}$ will be obtained from the first one simply by permutation of $\varepsilon_m$ and $\varepsilon_d$, as will be shown later.

We parameterize the branch-cut path $\Gamma_m$ using a parametric equation $\beta = (\varepsilon_m + it)^{1/2}$, so that $t = 0$ corresponds to the branch-cut point $\beta = \varepsilon_m^{1/2}$ and $f(\beta)$ becomes:

$$f(t) = \frac{ie_m \varepsilon_d \exp(ik_0 x \sqrt{\varepsilon_m + it})}{2\sqrt{\varepsilon_m + it} (\varepsilon_m \varepsilon_d - (\varepsilon_m + it) + \varepsilon_d \sqrt{-it})} .$$

The branch-cut path $\Gamma_m$ is chosen to run from the positive infinity to $t = 0$ along and just below the positive real axis $t$, where $f(\beta)$ is represented by $f_-(te^{i\pi/2})$, then around the branch point $t = 0$ on a circle, whose radius tends to zero, and finally from $t = 0$ to positive infinity.
just above the positive real axis $t$, where the integrand is represented by $f_+(t)$. Thus, the integral on the chosen branch-cut path is:

$$ \int_{\Gamma_m} f(\beta) d\beta = \int_0^\infty f_+(t) dt + \int_{\Gamma'_m} f_-(t \exp(2\pi i)) dt, \quad (7) $$

as the contribution from the infinitesimal circle path, on which $\exp(2\pi i) = 1$, is null. Clearly, $f_+(t) \neq f_-(t \exp(2\pi i))$ because for instance the square root term in the denominator of Eq. (6) $(-it \exp(2\pi i))^{1/2} = (-it)^{1/2} \neq (-it)^{1/2}$. Substituting Eq. (6) for $f(t)$ into Eq. (7) and performing some algebraic manipulations we obtain:

$$ \int_{\Gamma_m} f(\beta) d\beta = \frac{\varepsilon_i e^{i\pi/4} k^2_{sp}}{\varepsilon_m - \varepsilon_d} \int_0^{\infty} \exp(i k_0 x \sqrt{\varepsilon_m + it}) \sqrt{i} \frac{\exp(\sqrt{\varepsilon_m + it} \varepsilon_m + it)}{(\varepsilon_m + it) (\varepsilon_m + \varepsilon_d)} dt. \quad (8) $$

The integral in Eq. (8) was given and evaluated by numerical integration in Ref. [6]. We attempt to find closed-form solution of the integral. Let $\varphi(t)$ denoting the integrant in the right-hand side of Eq. (8), which has a simple pole at $t = i\varepsilon_m / (\varepsilon_m + \varepsilon_d)$ in the complex plane $t$ and two branch points at $t = i\varepsilon_m$ and $t = 0$. We choose a contour $C'$ containing a branch-cut path $\Gamma_m$ around $t = i\varepsilon_m$ and a branch-cut path $\Gamma'_m$ around $t = 0$, which runs just below and above the positive real axis $t$ and encircling $t = 0$ as shown in Fig. 3. Thus, the branch-cut integral along $\Gamma'_m$ equals $\int_0^\infty \varphi(t) dt$, as $\varphi(t)$ contains $t^{1/2}$ in the numerator so that $\varphi_-(te^{2\pi i}) = -\varphi_+(t)$. Expanding radius of the external circle of $C'$ to infinity and that of the internal circle around $t = 0$ to zero, using Cauchy’s formula and computing the residue around the pole, the integral in the right-hand side of Eq. (8) becomes:

$$ \int_0^\infty \varphi(t) dt = i\pi e^{i\pi/4} \left[ \frac{\varepsilon_m - ik_0^2}{\varepsilon_d} \right] - \frac{1}{2} \int_{\Gamma'_m} \varphi(t) dt. \quad (9) $$

To compute the branch-cut integral along $\Gamma_m$ we parameterize $\Gamma_m$ by $\tau^2 = \varepsilon_m + it$, such that the branch-point $t = i\varepsilon_m$ corresponds to the origin $\tau = 0$, and:

$$ \varphi(\tau) = -2ie^{-i\pi/4} \sqrt{\frac{\tau^2 - \varepsilon_m}{\tau^2 - k^2_{sp} / k^2_0}} e^{ik_0 \tau}. \quad (10) $$
The $\Gamma_m^{(r)}$ is chosen with the parameter $\tau$ running from 0 to $\infty$ with real and positive values, so that the corresponding integral path $\Gamma_m^{(r)}$ runs just below and above the positive real axis $\tau$ and encircling $\tau = 0$, as shown in Fig. 4.

As it is possible that $(\tau^2 - \epsilon_m)^{1/2}$ in the numerator of $\varphi(\tau)$ takes positive or negative signs, without loss of mathematic generality we take positive sign of $(\tau^2 - \epsilon_m)^{1/2}$ for $\varphi(\tau\epsilon^{1/2})$ on the segment in $\Gamma_m^{(r)}$ below the positive real $\tau$ axis and negative sign of $(\tau^2 \epsilon^{1/2} - \epsilon_m)^{1/2}$ for $\varphi(\tau)$ on the segment in $\Gamma_m^{(r)}$ above the positive real $\tau$ axis, such that $2^{1/2} (\epsilon \tau^2 - \epsilon_m) = -\varphi(\tau)$. The last integral on the right-hand side of Eq. (9) is then computed along the path $\Gamma_m^{(r)}$ as:

$$2 \int_{\tau} \varphi(\tau)d\tau = -4ie^{i\pi/4}k_0 \int e^{-ik} f_1(k) f_2(k) dk,$$

(11)

where $f_1(k) = \sqrt{k^2 - k_m^2}$, $f_2(k) = 1/(k^2 - k_{sp}^2)$, $s = -ix$, $k = k_0\tau$ and $k_m^2 = \epsilon_m k_0^2$. The integral in the right-hand side of Eq. (11), which is a Laplace transform, may be computed using the theorem of convolution as:

$$\mathcal{L}\{f_1(s)f_2(s)\} = \frac{1}{2\pi i} \int_{\alpha-i\infty}^{\alpha+i\infty} F_1(s) F_2(p-s) ds,$$

(12)

where the two Laplace transforms are $F_2(s) \equiv \mathcal{L}\{f_2(k)\}$ and:

$$F_1(s) \equiv \mathcal{L}\{f_1(k)\} = \frac{i\pi}{2s} \left( H_1(ik_m s) - Y_1(ik_m s) \right).$$

(13)

$H_1(ik_m s)$ is the Struve function, $Y_1(ik_m s)$ is the Bessel function of second kind and $\alpha$ is a real parameter larger than the real part of any singularities of the integrand, such that the Bromwich integral contour encircles all the singularities. When $s$ tends to zero $H_1(0) = 0$ and $Y_1(ik_m s)$ tends asymptotically to $-2/i\pi k_m s$, so that the second term in the right-hand side of Eq. (13) has a pole of second order at the origin. Thus, the Laplace transform in Eq. (11) is evaluated via the residue as:

$$\mathcal{L}\{f_1(s)f_2(s)\} = \lim_{s \to 0} \frac{d}{ds} F_2(p-s) = \frac{1}{2} \left( \int_0^\infty e^{i k x} \frac{dk}{k - k_{sp}} + \int_0^- \frac{e^{-ikx}}{k - k_{sp}} dk \right),$$

(14)

where $p = -ix$. The two integrals in Eq. (14) represent the composite superposition of plane waves of the positive and negative frequencies $k$, respectively, and include plane waves that are outgoing from and incoming to the line source. We keep only the outgoing waves, which are represented by the first integral in Eq. (14) for $x \geq 0$ and the second integral in Eq. (14) for $x \leq 0$. As the solution is now symmetric with respect to $x = 0$, we need to keep only the first integral in Eq. (14), which describes the waves propagating in the $+x$ direction and represented as:
\[
\int_0^\infty \frac{e^{ikx}}{k-k_{sp}} dk = e^{ik_{sp}x} E_i(i k_{sp}x), \quad x \geq 0, \tag{15}
\]

where the exponential integral is:
\[
E_i(z) = \int_z^\infty (e^{-t}/t) dt.
\]

Note that the solution represented in Eq. (15) of the branch-cut integral around \( \beta = \epsilon_m \) in Eq. (5) is independent of \( \epsilon_m \) itself, so that the solution of another branch-cut integral around \( \beta = \epsilon_d \) in Eq. (5) would be in the same forms with only permuting \( \epsilon_m \) to \( \epsilon_d \) in Eqs. (6)-(13). Putting together the solutions of the computed branch-cut integrals in Eqs. (5)-(15) and restore the constants that have been omitted for shortening the expressions, we obtain the final closed-form expression for the magnetic intensity component of the field at the interface \( y = 0 \) as:
\[
H_z(x) = i\omega \mu^* \mu \epsilon_m \epsilon_d \left( \frac{(\epsilon_m \epsilon_d)^{1/2}}{\epsilon_d^2 + \epsilon_m^2} + \frac{E_i(i k_{sp}x)}{2\pi(\epsilon_m + \epsilon_d)} \right) e^{ik_{sp}x}, \tag{16}
\]

where the first term is the conventional SPP, which is the pole’s contribution \( H_{sp} \) expressed in Eq. (4). The conventional SPP can propagate a length of \( L_{sp} = 1/(2 Im(k_{sp})) \), which is equal to 93 \( \mu m \) for Ag at \( \lambda = 1 \mu m \). The second term in Eq. (16) describes a transient mode, which is at first the SPP with the wave number \( k_{sp} \) and a complex-valued envelope \( E_i(i k_{sp}x) \), whose form depends on \( \epsilon_m \) and \( \epsilon_d \) in the optical frequency range of interest. The SPP transient envelope is shown in Fig. 5(a) for a silver slit in the air at wavelength \( \lambda = 1 \mu m \).

4. Discussion

Mathematically the exponential integral \( E_i(z) \) with complex valued \( z \) behaves as logarithm at small \( |z| \) when \( |\arg(z)| < \pi \). As \( |k_{sp}| - 1.02k_0, \theta = \arg(k_{sp}) - 0.001 \) for silver at \( \lambda = 1 \mu m \), at small \( x \) \( |\arg(i k_{sp}x)| < \pi \), the transient SPP envelope \( E_i(i k_{sp}x) \) has a real part decreasing as \( -\gamma/\ln(|k_{sp}|x) \), where the Euler constant \( \gamma \approx 0.5772 \), and a constant imaginary part \(-\pi/2 - \theta \). This approximation is good only for \( x < \lambda/20 \) for Ag at \( \lambda = 1 \mu m \), as shown in Fig. 5(a). When \( x \) increases up to \( x < 3-4 \lambda \) both the real and imaginary parts of the transient SPP envelope tended rapidly to zero. The phase of \( E_i(i k_{sp}x) \) tends asymptotically to \(-k_{sp}x - \pi/2 \) in the range of \( x < \lambda \), as shown in Fig. 5(b), so that for Ag at \( \lambda = 1 \mu m \) and in this distance range the transient SPP \( E_i(i k_{sp}x) \exp(i k_{sp}x) \) has a constant phase of \( \pi/2 \). In fact, the transient SPP is damped close to zero at this distance.
Fig. 5. (color online) a) Transient SPP envelope $E_1(ik_spx)$ with Euler constant $\gamma \approx 0.5772$ and
b) Phase of $E_1(ik_spx)$; for sliver slit in air with $\epsilon_m = -41.23 + 2.82i$ at $\lambda = 1 \mu m$.

In the plot of Eq. (15) shown in Fig. 6(a) we find that the decay of the transient SPP amplitude may be asymptotically approximated as $-\gamma - \ln(x)$, $1/x^{1/2}$ and $1/x$ in the distance ranges of approximately $x < \lambda/20$, $\lambda/20 < x < \lambda$, and $x > \lambda$, respectively, for Ag at $\lambda = 1 \mu m$.

Furthermore, the first two damping distance ranges enlarge with the increase of the wavelength and the conductivity of Ag, as shown in Fig. 6(b) for $\lambda = 9 \mu m$, at which the Ag is close to a perfect conductor. In fact, for a slit in a perfect conductor, which does not support SPP, the surface wave is the pure contribution of the radiated cylindrical wave [7,9], which can be described by the Hankel function with an asymptotic behavior also as $-\gamma - \ln(x)$ for small $x$, and as $1/x^{1/2}$ for large $x$. Thus, in the real metal case of Ag at $\lambda = 1 \mu m$ the decay rate of the transient SPP in the range $x < \lambda$ correspond approximately to the cylindrical wave decay. This distance range enlarges at $\lambda = 9 \mu m$. Beyond these two ranges the transient SPP drops faster as $1/x$ to zero. The damping of the transient SPP implies a loss of energy from the cylindrical wave pumped to the SPP. For even larger $x > 1 \ mm$ for the silver slit in air the exponential integral $E_1(ik_spx)$ will increase as exponential. However, this range of $x$ is beyond the reach of $\exp(ik_spx)$, which is damped to zero well before.

These results agree with Ref. [6], where the numerical plot of the Green function follows the $1/x^{1/2}$ decay for $\lambda = 9 \mu m$. At $\lambda = 0.633$, 1, and 3 $\mu m$ its decays as $1/x^{1/2}$ only at short distances of $x$ and then decays much faster for larger $x$. In fact our analytic solution includes all the three decay regimes depending on the distance region away from the source.
The transient SPP expressed in Eq. (15) contains the evanescent and the homogeneous modes. One can mathematically separate them [5,10] by separating the superposition integral in the left-hand side of Eq. (15) into the integral from 0 to \( k_0 \) and that from \( k_0 \) to \( \infty \). The latter integral describes the evanescent modes and may be expressed as:

\[
e^{ik_x x} E_1(i(k_{sp} - k_0)x). \tag{17}
\]

This term shows strong oscillation [10] due to the hard-cut in the Fourier spectrum at \( k_0 \). In fact, both homogeneous and evanescent modes oscillate along the \( x \)-axis. Their summation \( e^{ik_x x} E_1(i(k_{sp} x) \) shows, however, a monotonic and rapid decrease along the \( x \)-axis.

5. Conclusion

In conclusion, we have obtained a closed-form solution in a rigorous manner for the magnetic component of a TM polarized optical wave scattered by a sub-wavelength real-metal slit. The surface waves consist of a long-range SPP and a transient SPP with a complex-valued envelope of the exponential integral. The transient SPP includes also the contribution of homogeneous waves at the interface and can be considered physically as a cylindrical wave which is radiated from the dipole at the slit, takes the SPP propagation constant at the metal/dielectric interface and propagates with additional damping due to the loss of energy pumped to the SPP modes. This model based on a horizontal electric dipole or an equivalent magnetic current line source is approximate as the spatial extent of the slit is not taken into account. This closed-form result could be useful in the design of future plasmonic devices.

Appendix A

To obtain the Fourier domain Helmholtz equation from Maxwell’s equations in the equivalent magnetic current formalism we start with:

\[
\nabla \times \mathbf{E}_j = -\mathbf{j}_m^* - \mu_0 \partial_t \mathbf{H}_j; \tag{A1}
\]
\[ \nabla \times \mathbf{H}_j = - \varepsilon \frac{\partial}{\partial y} \mathbf{E}_j, \quad (A2) \]

For the TM polarization \( \mathbf{H}_j = (H_z, z), \mathbf{E}_j = (E_x, \hat{x}) + (E_y, \hat{y}) \) and \( \mathbf{J}_m^* = J_m^* \delta(x) \delta(y + d) \hat{z} \) we have:

\[ \partial_x (E_y)_j - \partial_y (E_x)_j = -J_m^* \delta(x) \delta(y + d) - \mu_0 \frac{\partial}{\partial z} (H_z)_j ; \quad (A3) \]

\[ \partial_y (H_z)_j = \varepsilon \frac{\partial}{\partial x} (E_x)_j ; \quad (A4) \]

\[ \partial_x (H_z)_j = -\varepsilon \frac{\partial}{\partial y} (E_y)_j. \quad (A5) \]

Consider the time harmonic of the form \( e^{i \omega t} \) and express the fields and current by the Fourier decomposition in domain of the spatial frequency associated with the spatial variable \( x \) with the Fourier decomposition as:

\[ \mathbf{A}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{\tilde{A}}(k_0 \beta, y) e^{ik_0 \beta x} d(k_0 \beta), \quad (A6) \]

where \( \mathbf{A} \) stands for either \( \mathbf{E} \), \( \mathbf{H} \) or \( \mathbf{J}_m^* \) and \( \mathbf{\tilde{A}} \) denotes their Fourier transforms. The Maxwell’s equations (A3), (A4) and (A5) can be expressed as:

\[ ik_0 \beta (\tilde{E}_y)_j - \partial_y (\tilde{E}_x)_j = -J_m^* \delta(y + d) + i \omega \mu_0 (\tilde{H}_z)_j ; \quad (A7) \]

\[ \partial_y (\tilde{H}_z)_j = -i \omega \varepsilon (\tilde{E}_x)_j ; \quad (A8) \]

\[ ik_0 \beta (\tilde{H}_z)_j = i \omega \varepsilon (\tilde{E}_y)_j. \quad (A9) \]

Putting Eqs. (A9) and (A8) into (A7) in order to obtain an equation for the magnetic field, we obtain Eq. (1).

**Appendix B**

General solution for the Helmholtz equation (1) in region 1 (metal) is

\[ \tilde{H}_1^{(b)} = C_1 e^{i \gamma_1 y} + C_2 e^{-i \gamma_1 y}. \quad (B1) \]

The particular solution is the 1D Green’s function

\[ \tilde{H}_1^{(p)} = \frac{-\omega \varepsilon J_m^*}{2\gamma_1} e^{i \gamma_1 |y + d|}. \quad (B2) \]

The complete solution for region 1 is therefore:

\[ \tilde{H}_1 = C_1 e^{i \gamma_1 y} + C_2 e^{-i \gamma_1 y} - \frac{\omega \varepsilon J_m^*}{2\gamma_1} e^{i \gamma_1 |y + d|}. \quad (B3) \]

For Region 2 (dielectric), the solution is general:

\[ \tilde{H}_2 = C_3 e^{i \gamma_2 y} + C_4 e^{-i \gamma_2 y}. \quad (B4) \]
By examining the complex-valued $\gamma_j^2 = k_0^2 (\epsilon_j - \beta^2)$ we find $\text{Im}\{\gamma\} > 0$. As the field must be bounded at infinity $y \to \infty$ in Region 1 and $y \to \infty$ in Region 2, we set the constants $C_1$ and $C_4$ to zero and obtain

$$\tilde{H}_1 = C_x e^{-i\gamma_x y} - \frac{\omega \epsilon_1 J_m^*}{2\gamma_1} e^{i\gamma_1 y + d};$$

(B5)

$$\tilde{H}_2 = C_x e^{i\gamma_x y}.$$  

(B6)

The continuity of the fields at the interface requires

$$\left. \tilde{H}_1 \right|_{y=0} = \left. \tilde{H}_2 \right|_{y=0};$$

(B7)

$$\left. \left( \tilde{E}_y \right) \right|_{y=0} = \left. \left( \tilde{E}_y' \right) \right|_{y=0}.$$  

(B8)

From (B7), we obtain:

$$C_5 = C_2 - \frac{\omega \epsilon_1 J_m^*}{2\gamma_1} e^{i\gamma_1 d}.$$  

(B9)

The tangent electric field $\tilde{E}_x$ is obtained from $\tilde{H}_2$ by Eq. (A8). Thus, the condition Eq. (B8) gives:

$$C_2 = \frac{\omega J_m^*}{2} e^{i\gamma_1 d} \frac{\gamma_1 + \gamma_2}{\epsilon_1 + \epsilon_2}.$$  

(B10)

Put the source at the interface ($d \to 0$), we obtain from Eqs. (B5) (B6) (B9) and (B10):

$$\tilde{H}_1 = -\frac{\omega J_m^*}{\gamma_1 + \gamma_2} e^{-i\gamma_x y}; \quad \text{and} \quad \tilde{H}_2 = -\frac{\omega J_m^*}{\gamma_1 + \gamma_2} e^{i\gamma_x y}.$$  

$\epsilon_1 \epsilon_2$ $\epsilon_1 \epsilon_2$